O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

ECEN/MAE 5513
Stochastic Systems
Fall 2011
Final Exam

PICK FIVE OUT OF ALL SIX PROBLEMS

Please specify which FIVE problems to be graded:
______, ______, ______, ______, and ______.

Name : ______________________________

E-Mail Address:____

<u>Problem 1</u>:
Let X be a Rayleigh random variable with $a = 0$. Find the probability that X will have values larger than its mode.

Please note from Homework Assignment #3:
<u>Problem 5</u>: Verify that the maximum value of $f_X(x)$ in which

$$f_X(x) = \frac{2}{b}(x-a)e^{-(x-a)^2/b}u(x-a)$$

for the Rayleigh density function occurs at $x = a + \sqrt{b/2}$ and is equal to $\sqrt{2/b}\exp(-1/2) \approx 0.607\sqrt{2/b}$. This value of x is called the *mode* of the random variable.

Problem 2:
A random variable X undergoes the transformation $Y = a/X$, where a is a real number. Find the density function of Y, $f_Y(y)$.

Please note $F_Y(y) = P\{Y \le y\} = P\{x \| Y \le y\} = \int_{\{x \| Y \le y\}} f_X(x)dx$

Problem 3:

The locations of hits of darts thrown at a round dartboard of radius *r* are determined by a vector random variable with components *X* and *Y*. The joint density of *X* and Y is uniform, that is

$$f_{X,Y}(x,y) = \begin{cases} 1/\pi r^2, & x^2 + y^2 < r^2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the densities of *X* and *Y*.

Problem 4:

For two statistically independent random variables X and Y show that

$$P\{Y \le X\} = \int_{-\infty}^{\infty} F_Y(x) f_X(x) dx$$

or

$$P\{Y \le X\} = 1 - \int_{-\infty}^{\infty} F_X(y) f_Y(y) dy \, .$$

Problem 5:

Consider random processes

$$X(t) = A\cos(\omega_0 t + \Theta)$$

$$Y(t) = B\cos(\omega_1 t + \Phi)$$

where A, B, ω_1, and ω_0 are constants, while Θ and Φ are statistically independent random variables each uniform on $(0, 2\pi)$.

a) Show that $X(t)$ and $Y(t)$ are jointly wide-sense stationary.
b) If $\Theta = \Phi$, show that $X(t)$ and $Y(t)$ are not jointly wide-sense stationary, unless $\omega_1 = \omega_0$.

Problem 6:

A Gaussian random process has an autocorrelation function

$$R_{XX}(\tau) = 6\frac{\sin(\pi\tau)}{\pi\tau}.$$

Determine a covariance matrix for the random variables $X(t)$, $X(t+1)$, $X(t+2)$ and $X(t+3)$.